Baseflow Recession Analysis: Testing the Nonlinear Reservoir Hypothesis
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INTRODUCTION
A streamflow recession curve exhibits behaviors which may be attributed to the relationship between aquifer formations and the associated groundwater outflow to the stream channel. Increased attention has focused on this relationship due to increased water flow during drought periods and its recognition as a significant component of the global freshwater budget. The purpose of this paper is to document experiments performed to evaluate the degree to which the following factors impact our ability to discern whether or not water levels act as linear reservoirs: 1) seasonality, 2) degrees of groundwater withdrawals in the watershed, 3) estimation of the numerical derivative in (3b) and (4) and the method used to fit the power law model to the lower envelope of the relationship between \( dQ/dt \) vs \( Q \).

DATA
Streamflow: 45 USGS Gage in New Jersey
Precipitation: National Climatic Data Center (NDCD)
GW Withdrawals: New Jersey Geologic Survey (NJS, 2011)

EXPERIMENTAL METHODOLOGY
Hall (1968) and Dooge (1983) suggested the use of a power law model to relate groundwater storage in an unconfined aquifer in direct hydraulic connection to a stream, \( S \), and the observed groundwater discharge to the stream, \( Q \).

\[
Q = aS^n \quad (1)
\]

where \( a \) and \( n \) are constants (attributed to Coutagne, 1948). For an unconfined aquifer, ignoring potential auxiliary effects, continuity can be written as

\[
\frac{dS}{dt} = \frac{1}{n}(Q - \alpha S^n) \quad (2)
\]

where \( \alpha \) represents infiltration. Combining (1) and (2) with \( dS/dt = 0 \) leads to

\[
\frac{dQ}{dt} = \frac{a}{n} \alpha Q^n \quad (3)
\]

where \( a \) and \( n \) are constants with \( b = \alpha - 1/(n+1) \) and \( a = \alpha^{1/(n+1)} \). Equation (3) was presented by Brutsaert and Nieber (1977) to graphically illustrate that on a recession plot of \( dQ/dt \) vs \( Q \), the parameter \( n \) is the intercept while \( b \) is the slope of the line to the streamflow recession data.

QUANTILE REGRESSION
Brutsaert and Nieber (1977) recommended fitting a lower envelope to the cloud of data created on a recession plot of \( dQ/dt \) vs \( Q \). Previous methods are generally not reproducible due to their subjective nature (Wang, 2011). Instead we propose the use of quantile regression which is a rigorous and reproducible method for fitting the lower envelope (Figure 2).

NUMERICAL DERIVATIVES
Our analysis requires estimation of numerical derivatives to evaluate (3). Numerical differentiation is “ill-posed” in a Hadamard sense (D’Amigo and Ferrigno, 1992) in that small measurement errors in a time series can result in large errors in derivative estimation. Streamflow derivative error can result from three potential sources: truncation error from the Taylor series approximations often used in practice, noise error (Liu et al., 2011) and systematic error in streamflow observations (D’Amigo and Ferrigno, 1992). To explore the relationship between the methods of estimation of the numerical derivative to be used in (3), we tested various estimators summarized in Table 1 to optimal GCV splines (Figure 3).

SEASONALITY
The Brutsaert approach recommended a lower envelope fit to characterize the power law relationship between \( dQ/dt \) and \( Q \) which represents groundwater contribution to streamflow. To characterize seasonal effects to the linear reservoir hypothesis, recession data from each site is separated into 5 categories: winter (Dec-Feb), spring (Mar-May), summer (June-Aug), fall (Sept-Nov) and annual datasets (Figures 5 & 6).

DEGREE OF WITHDRAWAL
To explore the relationship of groundwater withdrawals, an indicator variable, \( g \), is used which quantifies the extent of groundwater withdrawals as a ratio of overall streamflow

\[
\frac{dQ}{dt} = a^{(1-g)} Q^n \quad (4)
\]

Where \( a^{(1-g)} \) represents the monthly total groundwater withdrawals aggregated from HUC14 regions. We can combine (1) and (6) to obtain

\[
\frac{dQ}{dt} = a^{(1-g)} (Q - W) \quad (5)
\]

Figure 7: Estimates employing (5) for 1999-2007

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